Hypersonic Flow of a Diatomic Dissociating Gas with Power-Law Shock

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Self-similar solutions of Van Dyke's small-disturbance equations are found for the hypersonic flow in thermodynamic equilibrium of a diatomic dissociating gas. The gas is characterized in a thermodynamically consistent way by an equation of state that is shown to be a good approximation, valid over a considerable range of degrees of dissociation and densities, to Lighthill's ideal dissociating gas. As in the analogous case for a perfect gas, a shock appears whose coordinates satisfy a power-law of the form $y = y_0x^m$, and all dependent variables are functions essentially only of y/x^m. Results are obtained for two-dimensional and axisymmetric flows. A comparison with the corresponding solutions for a perfect gas with $\gamma = \frac{7}{5}$ shows qualitatively similar behavior, although there are significant quantitative differences.

I. Introduction

SIMILARITY solutions have played a major role in the development of hypersonic flow theory, since they represent one of the few means available to simplify the problem, which is inherently nonlinear. A number of such solutions are known,[1,2] such as the spherically expanding blastwave of Sedov[3] and Taylor,[4] or the small-disturbance, hypersonic steady flow with power-law shock of Velesko et al., and Lees and Kubota.[5]

In the present paper, the perfect gas solutions of Lees and Kubota are extended to diatomic dissociating gases. This extension is evidently of interest in applications to hypersonic flight such as occurs at re-entry into the earth's atmosphere. Dissociation, in this instance, is an inevitable consequence of the hypersonic approximation, which in turn forms the basis for these similarity solutions. Thermodynamic equilibrium of the flow behind the shock is assumed.

We consider therefore the inviscous, steady, hypersonic flow about a slender body, such as illustrated in Fig. 1. Two-dimensional, or, alternatively, axisymmetric flow is assumed. Ahead of the shock, the flow is uniform and, in the general case, may be partially dissociated.

It is convenient to introduce a reference length X_0 that characterizes the extent of the flow in the freestream direction for which the hypersonic small-disturbance approximation is appropriate. The shock angle (angle between shock and freestream direction) will be designated by $\sigma = \sigma(X)$, and is assumed to be positive for convenience.

Hypersonic small-disturbance theory, as formulated by Van Dyke,[6] can be viewed as the asymptotic theory that results when $\sigma_0 = \sigma(X_0) \ll 1$. The hypersonic approximation, i.e., $M_\infty \gg 1(M_\infty =$ freestream Mach number) is an easily derived consequence of the first inequality.

Designating U and V as the axial and transverse velocity components, respectively, and ρ, P, H, S as the density, pressure, specific enthalpy, and entropy, it is convenient to define new variables x, y, etc., by letting $x = X/X_0$, $\sigma_0 y = Y/X_0$, $\sigma_0^2\bar{u} = (U - U_\infty)/U_\infty$, $\sigma_0\bar{v} = V/U_\infty$, $\bar{\omega} = \rho/\rho_\infty$, $\sigma_0^2\bar{p} = (P - P_\infty)/\rho_\infty U_\infty^2$, $\sigma_0^2\bar{h} = (H - H_\infty)/U_\infty^2$. Here, $(\ )_\infty$ designates the condition upstream of the shock, and the exponent of the expansion parameters σ_0 in these definitions is chosen such that the new variables will turn out to be order one.

Omitting terms of order σ_0^2, the hypersonic, small-disturbance equations for plane ($j = 0$) or axisymmetric ($j = 1$) flow are, behind the shock,[6]

$$\partial\bar{\omega}/\partial x + \partial(\bar{\omega}\bar{v})/\partial y + j\bar{\omega}\bar{v}/y = 0 \tag{1a}$$

$$(\partial/\partial x + \bar{v}\partial/\partial y)\bar{v} + (1/\bar{\omega})\partial\bar{p}/\partial y = 0 \tag{1b}$$

$$(\partial/\partial x + \bar{v}\partial/\partial y)\bar{s} = 0 \tag{1c}$$

where $\bar{s}$ is proportional to the entropy. It is a well-known result of this theory that the axial velocity component does not appear explicitly in these equations; if needed it can be obtained from the x component of the momentum equation, or, more simply, from the energy equation

$$\bar{h} + \bar{u} + \tfrac{1}{2}\bar{v}^2 = 0 \tag{2}$$

Expanding in terms of the shock angle, and neglecting again terms of order σ_0^2, the boundary conditions at the shock are

$$(1 - 1/\bar{\omega})\sigma/\sigma_0 - \bar{v} = 0 \tag{3a}$$

$$(1 - 1/\bar{\omega})\sigma^2/\sigma_0^2 - \bar{p} = 0 \tag{3b}$$

$$\tfrac{1}{2}(1 + 1/\bar{\omega})\bar{p} - \bar{h} = 0 \tag{3c}$$

from conservation of momentum parallel to the shock together with the continuity equation, from momentum conservation normal to the shock, and from the Hugoniot equation, respectively.

In the case of a perfect gas, Eqs. (1) are known to possess a self-similar solution, provided the freestream Mach number and shock angle are further restricted to satisfy the condition $M_\infty \sin\sigma_0 \gg 1$, corresponding to the "strong shock" approximation of blast-wave theory. The same restriction will be made here, with the consequence that P_∞ and H_∞ are negligible compared with P and H.

Following Lees and Kubota,[5] we assume that the shock coordinates satisfy a power law of the form $Y/Y_0 = (X/X_0)^m$ as indicated in Fig. 1. [For a perfect gas with a ratio of the specific heats of $\frac{7}{5}$, and a convex body, m is known to be restricted to the range $\frac{2}{3} \leq m \leq 1$ (plane flow) and $\frac{1}{2} \leq m \leq 1$ (axisymmetric flow).[7]] Differentiating, and neglecting again terms of order σ_0^2 gives $\sigma/\sigma_0 = my_0x^{m-1}$ and therefore $my_0 = 1$. Hence, at the shock

$$y = (1/m)x^m \tag{4}$$

Transforming to new variables $\xi = x$ and $\eta = my/x^m$ results in the transformations

$$\partial/\partial x = \partial/\partial\xi - (m\eta/\xi)\partial/\partial\eta, \quad \partial/\partial y = (m/\xi^m)\partial/\partial\eta \tag{5}$$

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It follows that at the shock $\eta = 1$. Furthermore,

$$\sigma = \sigma_0 \xi^{m-1} \tag{6}$$

II. Similarity Solution for a Nonideal Gas

We show in this section that Eqs. (1) are satisfied by a similarity solution if the equation of state of the gas has the quite general form

$$h/p = \phi[\psi(s)/p] \tag{7}$$

where ϕ and ψ are arbitrary (differentiable) functions. We have introduced here the nondimensional enthalpy h, pressure p, and entropy s, normalized with respect to characteristic constants of the gas, by letting†

$$h = H/E_c,\ p = P/E_c\rho_c,\ s = ST_c/E_c$$

where E_c is a characteristic energy per unit mass (such as the dissociation energy of the gas), ρ_c a characteristic density (such as Lighthill's characteristic density of the ideal dissociating gas), and where similarly T_c is a characteristic temperature. We let $\psi(0) = 1$, without loss of generality.

Equation (7), which contains only canonically related variables of state, has the form of a "fundamental equation"[8] in the thermodynamic sense. The other variables of state can therefore be obtained directly by differentiation; in particular, the normalized density $\omega = \rho/\rho_c$ and temperature $\theta = T/T_c$ are, with the usual notation for thermodynamic derivatives,

$$1/\omega = (\partial h/\partial p)_s \tag{8a}$$

$$\theta = (\partial h/\partial s)_p \tag{8b}$$

It can be shown that (7) represents the most general form of the equation of state that allows a self-similar solution of the hypersonic small-disturbance equations. In the present analysis, we will simply show the sufficiency of this condition and in Secs. III–VI apply the results to an approximate but thermodynamically consistent description of a diatomic dissociating gas in local thermodynamic equilibrium. Within the limitations imposed by the approximation, ϕ and ψ will turn out to be universal functions of their arguments valid for a large class of diatomic gases (essentially the class that satisfies the requirements of Lighthill's ideal dissociating gas).

Letting now $z = \psi(s)$, it follows from (8a) and (7), with the prime denoting differentiation,

$$1/\omega = \phi(z/p) - (z/p)\phi'(z/p) = \chi(z/p) \tag{9}$$

which defines the function χ and shows that the density ratio ω depends only on z/p and therefore on h/p. For later use, we also note that from (7) and (9),

$$d\left(\frac{h}{p}\right) = \frac{\phi(z/p) - 1/\omega}{z/p}\, d\left(\frac{z}{p}\right) = \left(\frac{h}{p} - \frac{1}{\omega}\right) d\ln\frac{z}{p}$$

and hence

$$\ln\frac{z}{p} = \int \left(1 - \frac{1/\omega}{h/p}\right)^{-1} d\ln\frac{h}{p} \tag{10}$$

Equation (7) includes as a special case the equation of state of a perfect gas. This is easily verified by letting $\phi(z/p) = C(z/p)^{1/\gamma}$ and $\psi(s) = \exp(s)$, where γ is the ratio of the specific heats, and where C is a positive constant. (The choice of C is immaterial since it merely affects the zero point of the entropy.) Computing the density and temperature from (8a) and (8b) gives the desired result.

† Two different normalizations (barred and unbarred symbols) of the thermodynamic variables are needed. The barred variables in (1) are normalized by means of the freestream conditions. As shown, a posteriori, from the consistency of the solution, these variables are of order one. They are therefore useful in justifying the omission of the higher-order terms in the flow equations and boundary conditions. The unbarred quantities are normalized with respect to characteristic constants of the gas, and are needed because an equation of state independent of freestream conditions is wanted. The unbarred variables are not of order one, in general.

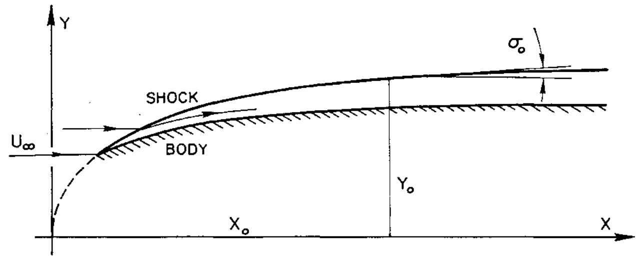

Fig. 1 Schematic diagram of hypersonic small-disturbance flow with power-law shock.

Returning again to the general case of (7), with arbitrary ϕ and ψ, we define $\bar{z}$ by $\sigma_0{}^2\bar{z} = zE_c/U_\infty{}^2$ and let

$$\bar{v} = \xi^{m-1}f_1(\eta) \tag{11a}$$

$$\bar{p} = \xi^{2(m-1)}f_2(\eta) \tag{11b}$$

$$\bar{z} = \bar{p}f_3(\eta) \tag{11c}$$

$$\bar{h} = \bar{p}f_4(\eta) \tag{11d}$$

$$1/\bar{\omega} = f_5(\eta) \tag{11e}$$

where the functions f_1–f_5 are independent of ξ. It is now easy to verify that Eqs. (1) are satisfied identically in ξ, if

$$f'_1f_5 + (\eta - f_1)f'_5 + (j/\eta)f_1f_5 = 0 \tag{12a}$$

$$(m - 1)f_1 - m(\eta - f_1)f'_1 + mf'_2f_5 = 0 \tag{12b}$$

$$2(m - 1)f_2f_3 - m(\eta - f_1)(f_2f_3)' = 0 \tag{12c}$$

$$f_4 - \omega_\infty\phi(\omega_\infty{}^{-1}f_3) = 0 \tag{12d}$$

$$f_5 - \omega_\infty\chi(\omega_\infty{}^{-1}f_3) = 0 \tag{12e}$$

where the argument η of the functions f_1–f_5 has been omitted for brevity, and where $\omega_\infty = \rho_\infty/\rho_c$ is the normalized freestream density. Equation (12d) is decoupled from the remaining equations in the set and serves to determine f_4, which occurs in the boundary conditions (13).

From Eqs. (3, 6, and 11), the boundary conditions at the shock (where $\eta = 1$) are

$$f_1(1) + f_5(1) - 1 = 0 \tag{13a}$$

$$f_2(1) - f_1(1) = 0 \tag{13b}$$

$$\tfrac{1}{2}f_1(1) + f_4(1) - 1 = 0 \tag{13c}$$

Since the function ψ does not appear in these equations, the solutions for f_1–f_5 are valid for a whole class of gases corresponding to different functions ψ in their equation of state. This is evident also directly from (7) and (8a) which show that the density is unaffected by the choice of ψ. Only in calculating the entropy, temperature, and degree of dissociation does ψ influence the results.

III. Approximation to Lighthill's Ideal Dissociating Gas

As Lighthill[9] has shown, an important class of diatomic homonuclear gases, including nitrogen and oxygen, can be described approximately by a single equation of state with three adjustable constants that are characteristic of the particular gas. These constants are: a characteristic energy for dissociation $E_c = D/2m_a$ (D = dissociation energy per molecule, m_a = mass of atom), a characteristic density ρ_c, and a characteristic temperature $T_c = D/k$ (k = Boltz-

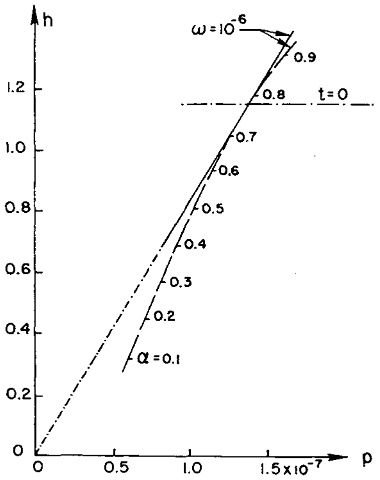

Fig. 2 Diagram illustrating the existence of a tangent to a line of constant density ratio ω. The solid line refers to the approximation (7), the dashed line to Lighthill's ideal dissociating gas. For other density ratios, the approximation is qualitatively similar. p = pressure ratio; h = enthalpy ratio; α = degree of dissociation; t = 0, tangent locus.

mann's constant).‡ Lighthill's equations, in terms of the normalized variables, for thermodynamic equilibrium, are

$$p = \omega\theta(1 + \alpha) \tag{14a}$$

$$h = (4 + \alpha)\theta + \alpha \tag{14b}$$

$$\alpha^2/(1 - \alpha) = (1/\omega)\exp(-1/\theta) \tag{14c}$$

$$s = 3\ln\theta + \alpha(1 - 2\ln\alpha) - (1 - \alpha)\ln(1 - \alpha) - (1 + \alpha)\ln\omega \tag{14d}$$

where α is the degree of dissociation (ratio of atom mass density to total mass density). The following relations, obtained from eliminating θ by means of (14c) are also useful:

$$p = \frac{\omega(1 + \alpha)}{\ln[(1 - \alpha)/\omega\alpha^2]}, \quad h = \frac{4 + \alpha}{\ln[(1 - \alpha)/\omega\alpha^2]} + \alpha \tag{15}$$

Lighthill's equation of state does not have the form (7), and does not permit a similarity solution of the hypersonic small-disturbance equations. However, a thermodynamically consistent approximation exists, which is of this form, and, as we will show, is a good approximation to Lighthill's equations over a considerable range of degrees of dissociation and density.

The nature of the approximation involved is illustrated in Fig. 2, where a curve of constant density ratio has been computed from Eqs. (15). The approximation used in this section essentially consists in replacing this curve by its tangent through the origin.§

More precisely, we need to show that all the thermodynamic variables that enter the flow equations and boundary conditions (i.e., s, h, p, ω) differ from their correct values by at most second-order terms in any neighborhood of the tangent locus [computed from (15) and designated by $t = 0$ in the figure].

To establish this, it is convenient to take s and h as the independent variables in the manner of a Mollier chart. In the remainder of this section, the primed variables will refer to Lighthill's ideal dissociating gas, the unprimed variables to the approximating gas.

‡ E.g., for nitrogen, based on the dissociation energy of 9.76 ev per molecule, E_c = 8.02 kcal/g, ρ_c = 130 g/cm^3, and T_c = 113,000°K following Lighthill. In applications of the ideal dissociating gas equations to problems of aerodynamics, the actual densities and temperatures are typically very small compared with the characteristic values.

§ We have relied on extensive numerical calculations to establish the existence of such a tangent in all cases considered.

Evidently, we can try to choose the functions ϕ and ψ such that at all points on the tangent locus

$$p = p', \ \omega = \omega' \qquad \text{at } t = 0 \tag{16}$$

For either gas, from (8a), $\omega = (\partial p/\partial h)_s$. Therefore $\omega = \omega'$ implies

$$(\partial p/\partial h)_s = (\partial p/\partial h)'_s \qquad \text{at } t = 0 \tag{17}$$

To prove the analogous equality for the derivatives of ω, we note that

$$(\partial\omega/\partial t)_{h/p} = 0, \ (\partial\omega/\partial t)'_{h/p} = 0 \qquad \text{at } t = 0 \tag{18}$$

The first of Eqs. (18), which is true identically in t, follows from (7) and (9); the second equation follows directly from the definition of the tangent locus. From the chain rule for partial derivatives, twice applied, and making use of (16) to (18),

$$\begin{aligned}
\left(\frac{\partial\omega}{\partial h}\right)_s &= \left(\frac{\partial\omega}{\partial t}\right)_{h/p}\cdot\left(\frac{\partial t}{\partial h}\right)_s + \left[\frac{\partial\omega}{\partial(h/p)}\right]_t\cdot\left[\frac{\partial(h/p)}{\partial h}\right]_s \\
&= \left[\frac{\partial\omega}{\partial(h/p)}\right]_t\cdot\left[\frac{1}{p} - \frac{h}{p^2}\left(\frac{\partial p}{\partial h}\right)_s\right] \\
&= \left[\frac{\partial\omega}{\partial(h/p)}\right]'_t\cdot\left[\frac{1}{p'} - \frac{h}{p'^2}\left(\frac{\partial p'}{\partial h}\right)_s\right] \\
&= \left\{\left(\frac{\partial\omega}{\partial t}\right)_{h/p}\cdot\left(\frac{\partial t}{\partial h}\right)_s + \left[\frac{\partial\omega}{\partial(h/p)}\right]_t\cdot\left[\frac{\partial(h/p)}{\partial h}\right]_s\right\}' \\
&= \left(\frac{\partial\omega}{\partial h}\right)'_s \qquad \text{at } t = 0
\end{aligned} \tag{19}$$

Equations (16, 17, and 19) show that the pressure ratio $p(s,h)$ and density ratio $\omega(s,h)$ of the approximating gas differ only by terms of $O(t^2)$ in a Taylor expansion about the tangent locus.

IV. Functions ϕ and ψ for a Dissociating Diatomic Gas

The most direct way to find ψ is to carry out the integration indicated in (10). For this purpose we need ω, which, as was noted, depends only on h/p, and can therefore be evaluated in particular on the tangent locus.

On this locus (where we can drop the distinction between primed and unprimed variables since they and their first

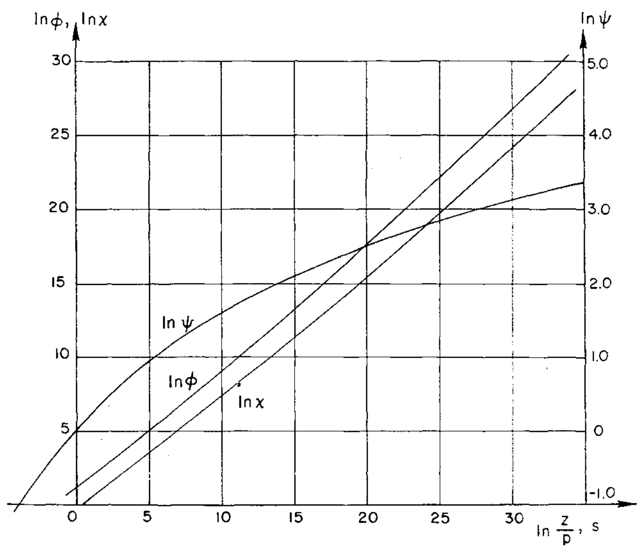

Fig. 3 Functions $\phi(z/p)$, $\chi(z/p)$, and $\psi(s)$ for dissociating diatomic gases. (For a perfect gas, the corresponding curves for ln ϕ and ln χ would be straight lines of slope $1/\gamma$.)

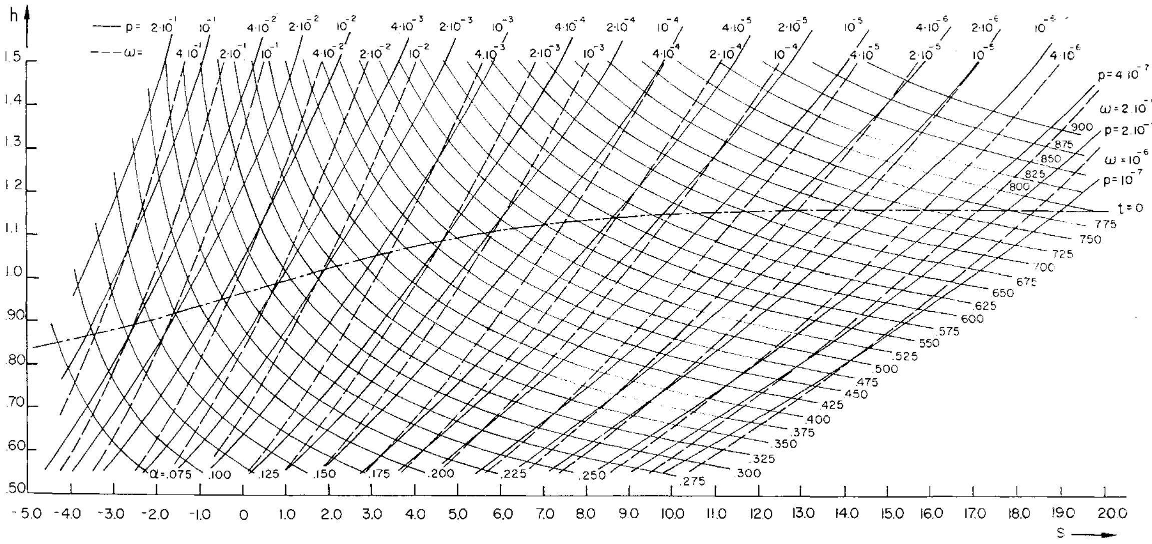

Fig. 4 Mollier diagram for dissociating diatomic gases satisfying the equation of state (7). h = ratio of enthalpy to dissociation energy; s = entropy ratio; α = degree of dissociation; p = pressure ratio; ω = density ratio; t = 0, tangent locus.

partial derivatives agree),

$$[\partial(h/p)/\partial\alpha]_\omega = 0$$

where p and h are given by (15). Carrying out the differentiation and solving for $1/\omega$, after some straightforward algebraic manipulations gives the relations

$$1/\omega = [\alpha^2/(1-\alpha)]\exp[(5-2\alpha-\alpha^2)/(1-\alpha)] \quad \text{at } t = 0 \qquad (20a)$$

$$\theta = (1-\alpha)/(5-2\alpha-\alpha^2) \quad \text{at } t = 0 \qquad (20b)$$

$$h/p = (1/\omega)[3+\alpha+1/(1-\alpha)] \quad \text{at } t = 0 \qquad (20c)$$

(20a) and (20c) give implicitly the needed relation between $1/\omega$ and h/p. Equation (10) has been integrated numerically, using a 5-point Lagrangian differentiation formula to transform to α as a new variable of integration. Computing the entropy s from (14d) and (20b) then gives $z = \psi(s)$.¶ The function is plotted in Fig. 3.

Similarly, since h/p in (7) depends only on z/p, the function ϕ can be determined from a calculation of h,p, and z on the tangent locus. It is again convenient to use the degree of dissociation α on the tangent locus as the independent variable. The calculation of χ is similar.

ϕ, ψ, χ are monotonically increasing functions of their arguments (Fig. 3). Insofar as Lighthill's equations are approximately valid for all diatomic homonuclear gases, ϕ, ψ, and χ are the same for all such gases, irrespective of their chemical composition.

Parenthetically we note that for a perfect gas the curves plotted in Fig. 3 for ln ϕ and ln χ would be straight lines of slope $1/\gamma$.

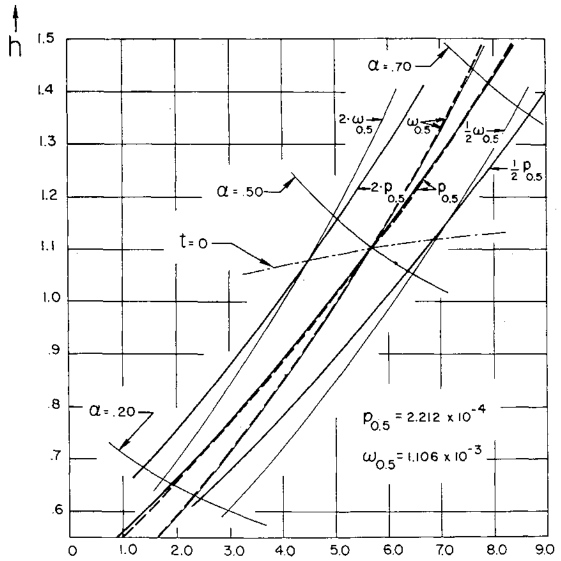

Fig. 5 Comparison of approximation (solid lines) with Lighthill's ideal dissociating gas (dashed lines). $p_{0.5}$ and $\omega_{0.5}$ designate lines of constant pressure and density ratio intersecting the tangent locus at α (degree of dissociation) = 0.5. The error in either pressure or density is less than 4% in the range $0.2 < \alpha < 0.7$.

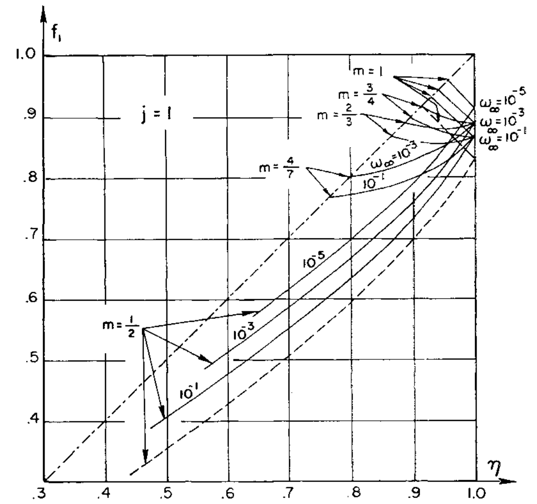

Fig. 6 Transverse velocity function f_1 vs η, for several values of the exponent m, and freestream density ratio $\omega_\infty = \rho_\infty / \rho_c$ (solid lines). Kubota's[10] results for a perfect gas with $\gamma = 7/5$, and $m = 1, 1/2$ are shown for comparison (dashed lines). The flow cannot be extended beyond the line $f_1(\eta) = \eta$. The shock front is at $\eta = 1$.

¶ The constant of integration in (10) is chosen in order to satisfy the normalization $\psi(0) = 1$. The zero-point of the entropy is arbitrary; we use the one employed by Lighthill.[9]

Table 1 Pressure coefficient C_p for power-law bodies (axisymmetric case): η_b = ratio of body radius to shock radius; $f_2(\eta_b) = C_p/2\sigma_0{}^2\xi^{2(m-1)}$; $j = 1$

	$m = 1$		$m = \frac{3}{4}$		$m = \frac{2}{3}$		$m = \frac{4}{7}$	
	η_b	$f_2(\eta_b)$	η_b	$f_2(\eta_b)$	η_b	$f_2(\eta_b)$	η_b	$f_2(\eta_b)$
$\omega_\infty = 10^{-1}$	0.9307	0.8977	0.8971	0.7215	0.8659	0.6318	0.7661	0.4933
$\omega_\infty = 10^{-3}$	0.9422	0.9143	0.9140	0.7395	0.8877	0.6508	0.8012	0.5146
$\omega_\infty = 10^{-5}$	0.9566	0.9355	0.9343	0.7615	0.9130	0.6734	0.8393	0.5389
ideal gas[a] $\gamma = \frac{7}{5}$	0.915	0.875	0.875	0.696	0.839	0.607	0.725	0.467

[a] After Kubota.

V. Range of Validity of the Approximation

The Mollier diagram for the gas with the equation of state (7) has been computed from (7) and (9) and is shown in Fig. 4. The diagram extends over a range for which the approximations made are appropriate.

The tangent locus (labeled by $t = 0$ in Fig. 4) on which the variables of state agree exactly with those of Lighthill's ideal dissociating gas, is found by computing ω and θ from (20a) and (20b) and substituting the results into (14b) and (14d).

The approximation is surprisingly good, for the range of variables shown in Fig. 4. This is illustrated in Fig. 5, where the approximation (solid lines) is compared with Lighthill's ideal dissociating gas (dashed lines). In the example chosen (lines of constant pressure and density ratio which intersect the tangent locus at $\alpha = 0.50$), the error in either pressure or density does not exceed 4%, for degrees of dissociation in the range $0.2 < \alpha < 0.7$.

VI. Solution of Flow Equations and Discussion of Results

The third-order, nonlinear system of Eqs. (12) is conveniently written as

$$\sum_j a_{ij}(\eta)f'_j(\eta) = b_i \qquad i,j = 1,2,3 \tag{21}$$

where, after some calculation necessitated by transforming barred variables,

$$\begin{aligned} &a_{11} = f_5,\ a_{12} = 0,\ a_{13} = (\eta - f_1)\frac{f_5}{f_3}\frac{d\ln\chi}{d\ln(z/p)} \\ &a_{21} = -m(\eta - f_1),\ a_{22} = mf_5,\ a_{23} = 0,\ a_{31} = 0 \\ &a_{32} = a_{21}f_3,\ a_{33} = a_{21}f_2,\ b_1 = -(j/\eta)f_1f_5 \\ &b_2 = -(m-1)f_1,\ b_3 = -2(m-1)f_2f_3 \end{aligned} \tag{22}$$

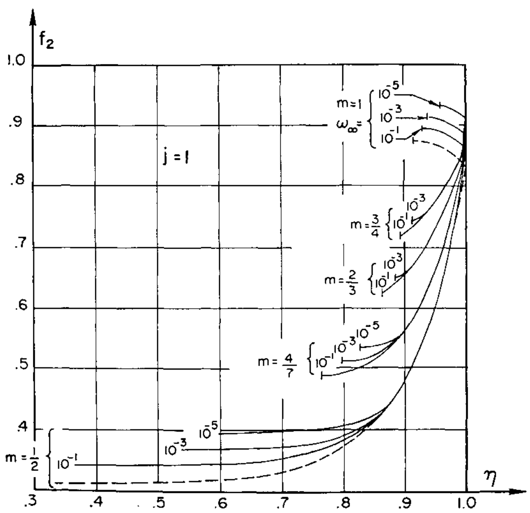

Fig. 7 Pressure function f_2 vs η; otherwise similar to Fig. 6.

Numerical solutions of (21), subject to the boundary conditions (13) were computed by means of the modified Euler (Heun's) method. Typical results for the transverse velocity function f_1 and the pressure function f_2 in the axisymmetric case ($j = 1$) are shown in Figs. 6 and 7.

The data for $m = 1$ and $\frac{1}{2}$ are compared with Kubota's[10] results for a perfect gas with a ratio of specific heats of $\frac{7}{5}$ (dashed curves). The results for the dissociating gas are seen to approach the perfect gas solutions as the freestream density ratio $\omega_\infty = \rho_\infty/\rho_c$ increases. This is to be expected since the degree of dissociation decreases with increasing density.

The solutions cannot be continued in a physically meaningful way below a value of $\eta = \eta_{\min}$ determined by

$$f_1(\eta_{\min}) = \eta_{\min} \tag{23}$$

At this value of η, the velocity becomes parallel to the line of constant η in the physical plane.[7]

In general, it is possible to choose arbitrarily one of the streamlines as the boundary of the flow. In particular, the line $\eta = \eta_{\min}$ defines a streamline that passes through the origin of the coordinate system, and, if taken as the boundary, defines an aerodynamic body whose coordinates satisfy a power law. On the surface of the body, $\eta = \eta_b = \eta_{\min}$.

η_b and the pressure coefficient

$$C_p = (P - P_\infty)/\tfrac{1}{2}\rho_\infty U_\infty{}^2 = 2\sigma_0{}^2\xi^{2(m-1)}f_2(\eta_b) \tag{24}$$

have been computed for axisymmetric power-law bodies, and are given in Table 1. For comparison, the corresponding results obtained for a perfect gas by Kubota are also listed.**

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** More extensive numerical data are contained in Ref. 11.